Bachelor of Science (B.Sc.) Semester—V (C.B.S.) Examination

MATHEMATICS

(M_9-Analysis)

Paper–1

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all **FIVE** questions.

(2) All questions carry equal marks.

(3) Questions **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT–I

1. (A) Obtain the Fourier series expansion of

$f(x) = x^2$ in $-\pi \le x \le \pi$;

and deduce that at x = 0

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots$$ 6

(B) Express f(x) = cosx as a Fourier sine series in the half range $0 < x < \pi$. 6

OR

(C) Find the Fourier series for the function f(x) defined by

$$f(x) = \begin{cases} 2, & -2 \le x < 0 \\ x, & 0 < x < 2 \end{cases}$$ 6

(D) Show that

$$\frac{1}{2}L - x = \frac{L}{\pi}\sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{2n\pi x}{L}\right), \; 0 < x < L.$$ 6

UNIT–II

2. (A) If P^* is a refinement of a partition P of [a, b], then prove that

$L(P, f, \alpha) \le L(P^*, f, \alpha)$,

where α is monotonically increasing function on [a, b]. 6

(B) If $f_1 \in R(\alpha)$ and $f_2 \in R(\alpha)$ on [a, b], then prove that $f_1 + f_2 \in R(\alpha)$. 6

OR

(C) If $f \in R(\alpha)$ on $[a, b]$, then prove that

$$|f| \in R(\alpha) \text{ and } \left|\int_a^b f d\alpha\right| \le \int_a^b |f|\, d\alpha .$$ 6

(D) Suppose F and G are differentiable functions on $[a, b]$, $F' = f \in R(\alpha)$ and $G' = g \in R(\alpha)$ on $[a, b]$. Then prove

$$\int_a^b F(x) g(x) dx = F(b)\, G(b) - F(a) G(a) - \int_a^b f(x) G(x) dx.$$ 6

UNIT–III

3. (A) If $f(z) = u + iv$ is analytic in a domain D, then prove that u, v satisfy the equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$ provided the four partial derivatives u_x, u_y, v_x, v_y exist. 6

(B) Find the analytic function f(z) of which the real part is $u = e^x(x \cos y - y \sin y)$. 6

OR

(C) If $u = x^2 - y^2$, $v = -\frac{y}{(x^2 + y^2)}$, then show that both u and v satisfy Laplace's equation, but $u + iv$ is not an analytic function of z. 6

(D) If u and v are harmonic in a region R, then prove that

$$\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$ is analytic in R. 6

UNIT–IV

4. (A) Determine the region R′ of w-plane into which the rectangular region R bounded by $x = 0$, $y = 0$, $x = 2$, $y = 3$ in z-plane is mapped under the map $w = z.e^{i\pi/4}.\sqrt{2}$. 6

(B) If there is only one invariant point p, then show that the bilinear transformation may be put in the form :

$$\frac{1}{w - p} = \frac{1}{z - p} + k, \quad \text{where } k = \frac{c}{a - cp}.$$ 6

OR

(C) Find the condition that the transformation $w = \frac{az + b}{cz + d}$ transforms the unit circle in w-plane into straight line of z-plane. 6

(D) Show that the transformation $w = \frac{2z + 3}{z - 4}$ transforms the circle $x^2 + y^2 - 4x = 0$ into the straight line $4u + 3 = 0$. 6

UNIT–V

5. (A) Obtain Fourier series for the function

$f(x) = 1$ on $(0, \pi)$. 1½

(B) Find the Fourier coefficient b_n for

$f(x) = x,\ 0 < x < 2\pi$. 1½

(C) Given that $f \in R(\alpha)$ on $[a, b]$. Then prove for positive constant c, $cf \in R(\alpha)$ on $[a, b]$. 1½

(D) State fundamental theorem of integral calculus. 1½

(E) Using polar form of Cauchy-Riemann equations,

show that $w = f(z) = z$ is analytic. 1½

(F) Prove that $u = (x - 1)^3 - 3xy^2 + 3y^2$ is a harmonic function. 1½

(G) Find the fixed points of the bilinear transformation

$$w = \frac{(2 + i)z - 2}{z + i}.$$ 1½

(H) Write the normal form of a bilinear transformation $w = \frac{z - 1}{z + 1}$; given that $z = i, -i$ are its fixed points. 1½